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# AN ALGORITHM FOR A NUMERICAL STUDY OF THE HYDRODYNAMICS and exchange of heat in a twisted channel of complex CROSS SECTION ON THE BASIS OF THE FINITE-ELEMENT METHOD 

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We propose the approximation and an algorithm for a numerical solution of the heat and mass transfer problem in a twisted channel of complex transverse cross section.

One method of intensifying the processes of heat and mass transfer in tubes and channels involves the twisting of the heat-carrying coolant in the inlet section with the aid of various turbulators. The resulting rotation of the flow in this case is retained over some initial segment of the channel, the length of this segment dependent on the shape of the transverse cross section, the initial kinetic energy of fluid rotation, its physical properties, the condition of the wall surfaces, etc. In a number of cases, the effect of these factors is so significant that the method by means of which the flow is twisted at the inlet to the channel becomes almost ineffective as a consequence of the rise in the level of energy expended on the pumping of the coolant and on its twisting, relative to the increased release of heat.

An alternate method of setting the flow in the rotation involves the use of channels with an extended twist (twisted channels), producing the effect of fluid rotation over the entire channel length. The efficiency achieved by such a method of intensifying the transfer processes, based on a series of complex experimental investigations into the hydrodynamics and heat and mass exchange in equipment with coiled tubes of oval and trefoil profiles, is demonstrated in [1-3]. At the same time, the literature contains no description of systematic approaches to the numerical modeling of transfer processes in twisted channels of complex cross section. A number of computational results $[4,5]$ have been obtained by the method of finite differences, while the computational algorithms constructed on the basis of this method have found their application limited to channels of classical cross-sectional shape (circular, rectangular).

The present study is devoted to an outline of the computational algorithm with which to study the processes of transfer in channels of complex cross section with longitudinal twist, using the finite-element method (FEM). By using this method is becomes possible significantly to expand the spectrum of modeled processes from the standpoint of diversity in the geometric shapes of the channel cross sections, to formulate a unique approach in the construction of discrete transfer-equation analogs, etc.

The processes of hydrodynamics and heat exchange in a channel of arbitrary transverse cross section for the case of the laminar flow of a viscous incompressible fluid, the absence of heat sources or sinks in the flow, and negligibly low viscous dissipation, are described by the following system of equations:

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$$
\begin{gather*}
\frac{\partial \bar{U}}{\partial t}+\frac{\partial\left(\bar{U}^{2}\right)}{\partial \bar{X}}+\frac{\partial(\bar{U} \bar{V})}{\partial \bar{Y}}+\frac{\partial(\bar{W} \bar{U})}{\partial \bar{Z}}=\frac{1}{\operatorname{Re}} \bar{\nabla}^{2} \bar{U}-\frac{\partial P}{\partial \bar{X}}+\bar{F}_{X} ; \\
\frac{\partial \bar{V}}{\partial t}+\frac{\partial(\bar{U} \bar{V})}{\partial \bar{X}}+\frac{\partial\left(\overline{V^{2}}\right)}{\partial \bar{Y}}+\frac{\partial(\bar{W} \bar{V})}{\partial \bar{Z}}=\frac{1}{\operatorname{Re}} \bar{\nabla}^{2} \bar{V}-\frac{\partial P}{\partial \bar{Y}}+\bar{F}_{Y} ; \\
\frac{\partial \bar{W}}{\partial t}+\frac{\partial(\bar{U} \bar{W})}{\partial \bar{X}}+\frac{\partial(\bar{V} \bar{W})}{\partial \bar{Y}}+\frac{\partial\left(\bar{W}^{2}\right)}{\partial \bar{Z}}=\frac{1}{\operatorname{Re}} \bar{\nabla}^{2} \bar{W}-\frac{\partial P}{\partial \bar{Z}}+\bar{F}_{Z} ;  \tag{1}\\
\quad \frac{\partial \bar{U}}{\partial \bar{X}}+\frac{\partial \bar{V}}{\partial \bar{Y}}+\frac{\partial \bar{W}}{\partial \bar{Z}} \cdots 0 ; \\
\operatorname{Pr}\left(\frac{\partial \bar{\Theta}}{\partial t}+\frac{\partial(\bar{U} \bar{\Theta})}{\partial \bar{X}}+\frac{\partial(\bar{V} \bar{\Theta})}{\partial \bar{Y}}+\frac{\partial(\bar{W} \bar{\Theta})}{\partial \bar{Z}}\right)=\frac{1}{\operatorname{Re}} \bar{\nabla}^{2} \bar{\Theta} ; \\
\{t>0 ; \bar{X}, \bar{Y} \in \bar{D}, \bar{Z}>0\},
\end{gather*}
$$

which has been written in dimensionless form for a fixed system of coordinates $\overline{\mathrm{X}}, \overline{\mathrm{Y}}, \overrightarrow{\mathrm{Z}}$.
In constructing the computational algorithm for the solution of system of equations (1) it is useful to make the transition to the curvilinear coordinate system $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$, the relative position of whose axes has been fixed for each channel cross section (Fig. 1). The relationship between the coordinates of the points, the components of the velocity vector, and the derivatives of some arbitrary scalar function $f(\bar{X}, \overline{\mathrm{Y}}, \overline{\mathrm{Z}}, \mathrm{t})$, in the transition to the new system of coordinates is expressed by the following equations:

$$
\begin{gather*}
\left.\left.\left|\begin{array}{l}
X \\
Y \\
Z
\end{array}\right|\left|\begin{array}{ccc}
\cos g & \sin g & 0 \\
-\sin g & \cos g & 0 \\
0 & 0 & 1
\end{array}\right| \right\rvert\, \begin{array}{l}
\bar{X} \\
\bar{Y} \\
\bar{Z}
\end{array}\right\} ; \\
\left\{\begin{array} { c } 
{ \overline { U } } \\
{ \overline { V } } \\
{ \overline { W } }
\end{array} \left|=\left|\begin{array}{ccc}
\cos g & -\sin g & 0 \\
\sin g & \cos g & 0 \\
0 & 0 & 1
\end{array}\right|\left\{\left.\begin{array}{l}
U \\
V \\
W
\end{array} \right\rvert\, ;\right.\right.\right.  \tag{2}\\
\left\{\begin{array} { c } 
{ \partial f / \partial \overline { X } } \\
{ \partial f / \partial \overline { Y } } \\
{ \partial f / \partial \overline { Z } }
\end{array} \left|=\left|\begin{array}{ccc}
\cos g & -\sin g & 0 \\
\sin g & \cos g & 0 \\
g^{\prime} Y & -g^{\prime} X & 1
\end{array}\right|\left\{\begin{array}{l}
\partial f i \partial X \\
\partial f i \partial Y \\
\partial f i \partial Z
\end{array}\right\},\right.\right.
\end{gather*}
$$

where $g=g(Z)$ is the parameter of longitudinal channel twisting; $g^{\prime}=\partial g / \partial Z$. Where its length is uniform $g=b d e Z /, b$ is the angle through which the channel cross section is turned over a segment of length $L$.

The original equations (1) in the $\mathrm{X}, \mathrm{Y}, \mathrm{Z}$ coordinate system, assuming the effect of diffusion transfer to the small for the substance in comparison to its convective transfer (parabolization of the problem) are written as follows:

$$
\begin{aligned}
\frac{\partial U}{\partial t}+\frac{\partial(\tilde{U} U)}{\partial X}+ & \frac{\partial(\tilde{V} U)}{\partial Y}+\frac{\partial(W U)}{\partial Z}+\frac{g^{\prime 2}}{\operatorname{Re}} U=\frac{1}{\operatorname{Re}}\left(\nabla^{2}+\tilde{V}^{2}\right) U- \\
& -\frac{\partial P}{\partial X}-\frac{2}{\operatorname{Re}} \tilde{\nabla} V+g^{\prime} V W+F_{X} ; \\
\frac{\partial V}{\partial t}+\frac{\partial(\tilde{U} V)}{\partial X}+ & \frac{\partial(\tilde{V} V)}{\partial Y}+\frac{\partial(W V)}{\partial Z}+\frac{g^{\prime 2}}{\operatorname{Re}} V=\frac{1}{\operatorname{Re}}\left(\nabla^{2}+\tilde{\nabla^{2}}\right) V
\end{aligned}
$$

$$
\begin{gather*}
-\frac{\partial P}{\partial Y}+\frac{2}{\operatorname{Re}} \tilde{\nabla} U-g^{\prime} U W+F_{Y} ;  \tag{3}\\
\frac{\partial W}{\partial t}+\frac{\partial(\tilde{U} W)}{\partial X}+\frac{\partial(\tilde{V} W)}{\partial Y}+\frac{\partial\left(W^{2}\right)}{\partial Z}=\frac{1}{\operatorname{Re}}\left(\nabla^{2}+\tilde{\nabla}^{2}\right) W- \\
- \\
-\frac{\partial P}{\partial Z}-g^{\prime} \tilde{\nabla} P+F_{Z} ; \\
\frac{\partial \tilde{U}}{\partial X}+\frac{\partial \tilde{V}}{\partial Y}+\frac{\partial W}{\partial Z}=0 ; \\
\operatorname{Pr}\left[\frac{\partial \Theta}{\partial t}+\frac{\partial(\tilde{U} \Theta)}{\partial X}+\frac{\partial(\tilde{V} \Theta)}{\partial Y}+\frac{\partial(W \Theta)}{\partial Z}\right]=\frac{1}{\operatorname{Re}}\left(\nabla^{2}+\tilde{V}^{2}\right) \Theta, \\
\{t>0 ; X, Y \in D, Z>0\} .
\end{gather*}
$$

Here, in order to reduce notation and to make the exposition of the computational algorithm more convenient, we have introduced the notation

$$
\begin{gathered}
\tilde{U}=U+g^{\prime} Y W ; \tilde{V}=V-g^{\prime} X W ; \nabla^{2}=\frac{\partial^{2}}{\partial X^{2}}+\frac{\partial^{2}}{\partial Y^{2}} ; \\
\tilde{\nabla}^{2}=Y \frac{\partial}{\partial X} \tilde{\nabla}-X \frac{\partial}{\partial Y} \tilde{\nabla} ; \tilde{\nabla}=g^{\prime 2}\left(Y \frac{\partial}{\partial X}-X \frac{\partial}{\partial Y}\right) .
\end{gathered}
$$

Let us note that the conditions of single-valuedness for system of equations (3) must be transformed in accordance with equalities (2).

Let us now turn to a description of the approach involved in the solution of this class of problems, which is a generalization of the algorithm for the solution of hydrodynamic problems in channels of complex cross section, as outlined in [6]. We will assume that $P$ is represented in the form of the sum of the functions $p(X, Y, t)$, i.e., the pressure in the cross-sectional plane of the channel and $\mathrm{\rho}(\mathrm{Z}, \mathrm{t})$, the pressure averaged over the channel cross section. Having made the regions D discrete with triangular elements and using these as a base to construct the control volumes [6,7], we will examine the following model transfer equation to choose the approximation of the unknown element functions:

$$
\begin{gather*}
\tilde{U}_{0} \frac{\partial f}{\partial X}+\tilde{V}_{0} \frac{\partial f}{\partial Y}=\frac{1}{\operatorname{Re}_{X_{0}}} \frac{\partial^{2} \dot{\partial}}{\partial X^{2}}+\frac{1}{\operatorname{Re}_{Y_{0}}} \frac{\partial^{2} f}{\partial Y^{2}}+ \\
+\frac{2}{\operatorname{Re}_{X Y_{0}}} \frac{\partial^{2} f}{\partial X \partial Y}+Q_{f_{0}}, \tag{4}
\end{gather*}
$$

written out for some element. Equation (4) describes the convective-diffusion transfer of a substance $f$ with consideration given to the anisotropy of its properties with respect to the $\mathrm{X}, \mathrm{Y}$ coordinates under the conditions for the determination of $\tilde{\mathrm{U}}, \tilde{\mathrm{V}}$, as well as of the coefficients of diffusion transfer in the operator $\tilde{\nabla}^{2}$ at the center of mass of the triangle (subscript 0 ) represents an approximate analog of the steady-state equations of system (3), with the initial term $\mathrm{Q}_{\mathrm{f} 0}$ containing the terms of the equations which cannot be expressed through the gradient in f .

We will construct the approximation of the unknown functions $U, V, W, \Theta$ on the element as the superposition of the partial solutions for (4):

$$
\begin{equation*}
\tilde{f}=Q_{i_{0}} \eta+\alpha+\beta \xi_{1}+\gamma \xi_{2} \quad(\tilde{f}=U, V, W, \Theta), \tag{5}
\end{equation*}
$$

where

$$
\xi_{1}=\frac{1}{\operatorname{Re}_{X_{0}} \tilde{U}_{0}}\left(\exp \left\{\operatorname{Re}_{X_{0}} \widetilde{U}_{0}\left(X-X_{0}\right)\right]-1\right) ;
$$

$$
\begin{gathered}
\xi_{2}=\frac{1}{\operatorname{Re}_{Y_{0}} \tilde{V}_{0}}\left(\exp \left[\operatorname{Re}_{Y_{0}} \tilde{V}_{0}\left(Y-Y_{0}\right)\right]-1\right) ; \\
\eta=\frac{1}{\operatorname{Re}_{\Delta}+1}\left(\operatorname{Re}_{\Delta} \varphi+\varphi\right) ; \\
\psi=-\frac{\operatorname{Re}_{\Sigma}}{2 \tilde{W}_{0}^{2}}\left[\tilde{U}_{0}\left(Y-Y_{0}\right)-\left.\tilde{V}_{0}\left(X-X_{0}\right)\right|^{2} ;\right. \\
\left.\varphi=\mid \tilde{U}_{0}\left(X-X_{0}\right)+\tilde{V}_{0}\left(Y-Y_{0}\right)\right\} / \tilde{W}_{0}^{2} ; \\
\operatorname{Re}_{\Sigma}=\tilde{W}_{0 /}^{2}\left(\tilde{U}_{0}^{2} / \operatorname{Re}_{Y_{0}}+\tilde{V}_{0}^{2} / \operatorname{Re}_{X_{0}}+2 \tilde{U}_{0} \tilde{V}_{0} \cdot \operatorname{Re}_{X Y_{0}}\right) ; \tilde{W}_{0}=V / \overline{\tilde{U}_{0}^{2}+\tilde{V}_{0}^{2}} ;
\end{gathered}
$$

$\operatorname{Re}_{\Delta}=\operatorname{Re}_{\Sigma} \breve{W}_{0} d_{\Delta}$ is the analog of the Reynolds grid number determined from the characteristic dimension $d_{\Delta}$ of the element (for example, the square root of its area) and velocity $\bar{W}_{0} ; \alpha, \beta, \gamma$ are the unknown approximation parameters determined from the condition that the functions $\bar{f}$ are equal to the values of $f$ at the nodes of the triangular element.

It is easy to see that $\xi_{1}$ and $\xi_{2}$ are particular solutions of the uniform equation (4) and that $\eta$ is particular solution of (4) with a single source. The functions $\varphi$ and $\psi$ have been selected as the solutions (when $\mathrm{Q}_{\mathrm{f} 0}=1$ ) of the convective and diffusion parts of (4), respectively, with the weight factor $\mathrm{Re}_{\Delta}$ regulating the extent to which convection or diffusion exert influence in the overall transfer process to the element $\left(\eta \rightarrow \varphi\right.$ as $\operatorname{Re}_{\Delta} \rightarrow \infty ; \eta \rightarrow \psi$ as $\operatorname{Re}_{\Delta} \rightarrow 0$ ).

In a direct examination of the equations in system (3) we determine $\alpha, \beta$, and $\gamma$ in (4) to the functions of $Z$, $t$, and under the condition that $p(X, Y, t)$ changes linearly at each element, we will have

$$
\begin{align*}
Q_{U_{0}} & =-\frac{\partial p}{\partial X}-\frac{2}{\operatorname{Re}}(\tilde{\nabla} V)_{0}+g^{\prime}(V W)_{0}+F_{X_{0}} \\
Q_{V_{0}} & =-\frac{\partial p}{\partial Y}+\frac{2}{\operatorname{Re}}(\tilde{\nabla} U)_{0}-g^{\prime}(U W)_{0}+F_{Y_{0}}  \tag{6}\\
Q_{W_{0}} & =-\frac{\partial \bar{p}}{\partial Z}-g^{\prime}(\tilde{\nabla} p)_{0}+F_{Z_{0}} ; Q_{\Theta_{0}}=0 .
\end{align*}
$$

Formation of the resolving of equations relative to the nodal values of the unknown functions is accomplished by integration [7] of the equations in system (3) over all control volumes of region $D$ in conjunction with expressions (5) and (6), and approximation of the derivatives with respect to time and the coordinate Z by means of an implicit difference scheme.

The sequence of calculations described [6] for this class of problems undergoes the following changes (we will subsequently use the notation from [6]). The determination of the nodal values for $U^{\prime}, V^{\prime}$, and $W^{\prime}$ is accomplished through iteration solution of system (3) owing to the interconnectedness of its equations, with the source terms in (6) calculated from the values of the corresponding functions in the previous iterations. The pressure-gradient correction factors $\partial \bar{\rho}^{1} / \partial Z$ are found, as in [6], from the condition of a constant rate of flow through the transverse section of the channel. In determining the pressure correction factors $\mathrm{p}^{\prime \prime}$ we represent the functions $\mathrm{U}^{\prime \prime}, \mathrm{V}^{\prime \prime}$, and $\mathrm{W}^{\prime \prime}$ at the elements in the form

$$
\begin{align*}
& U^{\prime \prime}=-C \frac{\partial(\delta p)}{\partial X}+f_{U}\left(X, Y, Z, t, U^{*}, V^{*}, W^{*}, p^{*}\right) \\
& V^{\prime \prime}=-C \frac{\partial(\delta p)}{\partial Y}+f_{V}\left(X, Y, Z, t, U^{*}, V^{*}, W^{*}, p^{*}\right)  \tag{7}\\
& W^{\prime \prime}=-C g^{\prime} \tilde{\nabla}(\delta p)+f_{W}\left(X, Y, Z, t, U^{*}, V^{*}, W^{*}, p^{*}\right)
\end{align*}
$$

Here $C=\left(1 / \Delta t+W_{0} / \Delta Z+g^{2} / R e\right)^{-1} ; \Delta t$ and $\Delta Z$ represent the intervals of the difference grid for the variables $t$ and $Z$, respectively; $f_{U}, f_{V}$, and $f_{W}$ are the approximation expressions at the element, derived through utilization of (5), where the coefficients $\alpha, \beta$, and $\gamma$ and the source terms in (6) are calculated from the values of $\mathrm{U}^{*}, \mathrm{~V}^{*}$, and $\mathrm{W}^{*}$ and $\mathrm{p}^{*} ; \mathrm{U}^{*}, \mathrm{~V}^{*}$, and $\mathrm{W}^{*}$ are the nodal values of the velocity vector component, corresponding to the pressure $p^{*}$. Let us note that the representation of $W^{\text {w }}$ in the form of (7) is undertaken after the addition of the term $\mathrm{g}^{2} \mathrm{~W} / \mathrm{Re}$ to both sides of the third of the equations in system (3).


Fig. 1. Configuration of the theoretical area, the boundary conditions, and the distribution of the velocity component $U$ along the cross-sectional diagonal; solid line) $\mathrm{Z} / \mathrm{Re}=0.005$; dashed line) 0.02 .


Fig. 2. Distribution of the velocity component $V$ along the diagonal of the cross section; solid line) $\mathrm{Z} / \mathrm{Re}=0.005$; dashed line) 0.02 .

Substitution of (7) into the fourth equation of system (3) (the continuity equation) yields the elemental analog of the Poisson equation for the pressure $\delta \mathrm{p}$ :

$$
\begin{equation*}
C\left(\nabla^{2}+\tilde{\nabla}^{2}\right) \delta p=\frac{\partial\left(\tilde{U}^{\prime}+\tilde{f}_{U}\right)}{\partial X}+\frac{\partial\left(\tilde{V}^{\prime}+\tilde{f}_{V}\right)}{\partial Y}+\frac{\partial W}{\partial Z} \tag{8}
\end{equation*}
$$

where $\bar{f}_{\mathrm{U}}$ and $\overline{\mathrm{f}}_{\mathrm{V}}$ are introduced in analogy with $\tilde{\mathrm{U}}$ and $\tilde{\mathrm{V}}$.
The integration of (8) and the summation of the derived equalities over all the elements of $D$ leads to a system of algebraic equations relative to the nodal values for the pressure $\delta \mathrm{p}$. The new distribution of $\mathrm{p}^{*}$ is found in the same way as in [6]. Let us note that in this case the difference in the sequence of calculations from that presented in [6] involves the need to correct the correction factors of the pressure gradient $\partial \mathrm{p}^{\prime \prime} / \partial \mathrm{Z}$ at each step in the determination of the field $\mathrm{p}^{*}\left(\mathrm{p}^{\prime \prime}\right)$ owing to its influence on the field of the velocity $W$ (the term $g^{\prime} \tilde{\nabla} p$ in the equation for $W$ ).

The general iteration process for the theoretical channel cross section is continued until the specified accuracy is achieved for all components of the velocity vector. The solution of the energy equation is included in the iteration process, provided that
the fields of velocity and temperature are interrelated (for example, consideration of free convection within the channel), or it is carried out after determination of $\mathrm{U}, \mathrm{V}$, and W .

The approach outlined above and the computational algorithm have been realized in the form of a complex of applied programs for the solution of problems related to hydrodynamics and heat exchange in twisted and helical channels of complex cross section, in a laminar fluid flow regime.

In order to test the complex of programs and the computational algorithm, we solved the problem pertaining to the flow of the fluid in the initial segment of a square channel (we used a grid consisting of 1152 elements and 625 nodes) for the case in which $\mathrm{Re}=500, \mathrm{~b}=0$ (a straight channel). The divergence of the derived data from the known analytical [8], numerical [9] solutions and the experimental data from [10] did not exceed $2 \%$. The nonsteady problem of calculating the fields of velocity and temperature in the segment of hydrodynamic and thermal stabilization in a channel of quadratic cross section was solved in the formulation presented in [4,5]. The calculations were carried out for the case of $\operatorname{Re}=243, \operatorname{Pr}=0.7$, and for a dimensionless coefficient of twisting (the ratio of the length over which the channel is twisted by $\pi$ radians to the half-width of the channel) $\mathrm{H}=2.5$. We assume the longitudinal pressure gradient to be equal to that given in [4]. The theoretical data for the local Nusselt number are found to be in good agreement with the known numbers [5]. Unfortunately, no data exist in [4,5] which are convenient for purposes of comparing the theoretical values of the velocity vector components in twisted channels.

We examine the problems of hydrodynamics and heat exchange in the initial segment for the case of coolant flow ( $\mathrm{Re}=500$, $\operatorname{Pr}=0.7$ ) in a square channel for various values of the parameter H with uniform distribution of velocity at the inlet and with thermal boundary conditions of the first kind at the walls (see Fig. 1).

Figures 1 and 2 show graphs of the change in the velocity vector components $U$ and $V$ along the diagonal ( $0 \leq \mathrm{X}=\mathrm{Y} \leq 0.5$ ) of the transverse cross section of the channel for various values of Z and H . We can see from the figures that in the absence of channel twisting $(\mathrm{H}=\infty)$ the transverse flow of the fluid through the channel sections is generated by the reshaping of the profile for the longitudinal velocity component over the length and it characterizes the flow of the fluid away from the walls to the core of the flow. As the parameter H is reduced, the fluid flow pattern through the transverse cross sections of the channel changes because of the onset of rotational motion in the flow. Even when $\mathrm{H}=100$, such motion (even though weak) arises in this case at some distance from the inlet section ( $\mathrm{Z} / \mathrm{Re} \geq 0.01$ ). We can see from the figures that in all of the channel sections the rotational motion of the fluid is most intense in the circular region near the walls. With increasing distance from the entry into the channel, the intensity of rotation in the circular zone for cases of low values for the parameter $\mathrm{H}(\mathrm{H} \leq 25)$ diminishes, and the transverse motion of the fluid in the core of the flow in this case assumed the nature of rotation. However, for large values of $\mathrm{H}(25<\mathrm{H} \leq$ $\infty)$ as Z increases there is a change from flow toward the axis of the channel to rotational motion with rising intensity.

Figure 3 shows the theoretical values of the local Nusselt number

$$
\begin{equation*}
\mathrm{Nu}=\frac{\partial \Theta|\partial n|_{\mathrm{w}}}{\Delta \Theta}, \Delta \Theta=\Theta_{\mathrm{w}}-\tilde{\Theta} \tag{9}
\end{equation*}
$$

(here n is the external normal to the boundary of region $\mathrm{D} ; \Theta_{\mathrm{w}}$ and $\bar{\Theta}$ represent the temperature of the wall and the mean-mass temperature of the fluid, respectively) for various values of the parameter H at various sections of the channel. We can see that at the cold (Fig. 3a) and hot (Fig. 3b) walls in the initial segment of the channel the heat-transfer coefficients differ markedly from one another. Their quantitative differences are explained by the entry into the inlet section of a fluid exhibiting the temperature


Fig. 3. Distribution of the Nu number on the cold (a) and hot (b) walls of the channel (solid line, $\mathrm{Z} / \mathrm{Re}=$ 0.005 ; dashed line, 0.02 ): 1) $\mathrm{H}=\infty$; 2) 100 ; 3) 50 ; 4) 25 ; 5) 12.5 .
of the cold wall, while the qualitative differences can be explained by the hydrodynamics of the coolant flow. Thus, in the immediate area of the corners of the channel at the cold $(X \rightarrow 0.5)$ and hot $(Y \rightarrow 0.5)$ walls, the drop in heat transfer with a reduction in H is governed by the reduction in the absolute value of the longitudinal component of the velocity vector and, consequently, by the increase in these zones of the role played by the diffusion transfer of enthalpy relative to convective transfer.

With increasing distance from the corners (to the left along the cold wall and downward along the hot wall) this rise in the heat-transfer coefficient is governed by the influence of the secondary flow in the circular portion which carries the fluid away from the hot wall to the cold, and reverse (the transverse motion in the circular region is accomplished in counterclockwise direction). The extent to which the absolute values of the Nu numbers at the cold wall exceed those of the hot wall for the channel sections being discussed here can be explained by the differing influence exerted by the terms in formula (9) on the theoretical values of $N u$. Thus, in the given case the decisive effect on the value of the $N u$ number is exhibited by the quantity $\Delta \Theta$, which is considerably smaller for the cold wall than for the hot wall. With increasing distance from the entry into the channel, the quantity $\Delta \theta$ for walls with different temperatures come close to one another and it is the local flow of heat at the walls that begins to have a decisive effect on the Nu number.

## NOTATION

$t=w_{0} \tau / d_{e}$, time; $w_{0}$, characteristic coolant flow velocity; $d_{e}$, hydraulic diameter of channel cross section; $\bar{U}=\mathbb{a} / w_{0}, \nabla=$ $\nabla / w_{0}$, and $\bar{W}=W / w_{0}$, dimensionless components of the velocity vector; $\bar{\theta}=\left(T-T_{0}\right) / \Delta T$, dimensionless temperature; $T_{0}$ and $\Delta T$, scale values for temperature; $\bar{X}=\mathbb{X} / d_{e} ; \bar{Y}=Y / d_{e} ; \bar{Z}=Z / d_{e} ; P=\Pi /\left(\rho w_{0}^{2}\right) ; \Pi$, pressure; $\operatorname{Re}=w_{0} d_{e} / \nu$, Reynolds number; $\operatorname{Pr}=$ $\nu / a$, Prandtl number; $\nu$ and $a$, kinematic viscosity and thermal diffusivity of the fluid; $\bar{F}_{X}, \overline{\mathrm{~F}}_{\mathrm{Y}}, \overline{\mathrm{F}}_{\mathrm{Z}}$, components of the total vector of mass forces active in the flow; $\bar{D}$, region of transverse channel section; $\nabla^{2}=\partial^{2} / \partial \boldsymbol{X}^{2}+\partial^{2} / \partial \bar{Y}^{2}+\partial^{2} / \partial Z^{2}$.

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